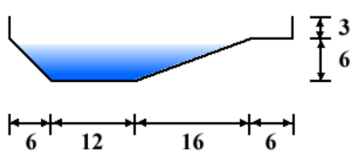
*Here are some extra practice problems I have collected for my MFE I recitation students. Most of these are seminar questions from Math 150 at Simon Fraser University (SFU). Any questions from SFU are noted. The last problem in each section addresses a cool mathematical property, definition, or fact relevant to the section (usually these problems will be quite hard for calculus students). This document will be periodically updated over the Fall 2025 term.*

# 1. Functions Review

1. (SFU) Consider the function $f(x) = \frac{|x^3+3x|}{x}$.
   (a) Find the domain of $f$.
   (b) Simplify the expression $\frac{|x^3+3x|}{x}$ for $x < 0$.
   (c) Let $g(x)$ be the result from part (b). What is the domain of $g$? Are $f$ and $g$ the same functions?
2. (SFU) A swimming pool is 20 ft wide, 40 ft long, 3 ft deep at the shallow end, and 9 ft deep at its deepest point. A cross-section is shown in the figure below.
   (a) Express the volume of the water in the pool as a function of height $h$ of the water above the deepest point *Hint: the volume will be a piecewise-defined function.*
   (b) Find the domain and range of the function found in part (a).




3. (SFU) Let $L$ be the line through the points $(0, -2)$ and $(2, 4)$ and $f$ be the function given by $f(x) = x^2 - 2x + 2$.
   (a) Find the equation of the line $L$. Determine whether the points $(-1, -3)$ and $(\frac{7}{3}, 5)$ lie on $L$.
   (b) Determine whether the points $(3, 2)$ and $(\sqrt{3} + 1, 4)$ are on the graph of $f$.
   (c) Sketch the graph of $L$ and $f$ on the same coordinate axes.
   (d) Find the intersection points of $L$ and the graph of $y = f(x)$.
4. (SFU) Let $A, B \subseteq \mathbb{R}$ be subsets of the real numbers. Suppose $f$ has domain $A$ and $g$ has domain $B$.
   (a) What is the domain of $f + g$?

(b) What is the domain of $f, g$?

(c) What is the domain of $f/g$?

5. Let $f$ be a function on some subset $A$ of the real numbers $\mathbb{R}$ ($f\colon A \to \mathbb{R}$, where $A \subseteq \mathbb{R}$). Define the pre-image of a set $B \subseteq \mathbb{R}$ under $f$ as

$$f^{-1}(B) = \{x \in A : f(x) \in B\}.$$

Let $A, B \subseteq \mathbb{R}$. Prove the following claims:

(a) $f(A \cup B) = f(A) \cup f(B)$.

(b) $f(A \cap B) \subseteq f(A) \cap f(B)$.

(c) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.

(d) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.

(e) Provide a counterexample to the claim $f(A \cap B) = f(A) \cap f(B)$.

## 2. Inverse Functions and Logarithms

1. (SFU) Let $g(x) = \ln\big(x + \sqrt{x^2 + 1}\big)$.

(a) Find the domain of $g$.

(b) Show that $g(x)$ is an odd function (i.e. $g(-x) = -g(x)$).

(c) Why is $g(x)$ a one-to-one (injective) function?

(d) Find the inverse of $g(x)$.

2. (SFU) Solve $\ln(x^2 - 1) = 3$ for $x$.

3. Define $f(x) = \frac{\ln\left(x^2\right)}{x^4 - 1}$.

(a) Find the domain of $f$.

(b) Determine, in interval notation, the set of all $x$ such that $f(x) \geq 0$.

4. (SFU) Suppose $f$ is a one-to-one (injective) function whose *inverse* is given by the formula

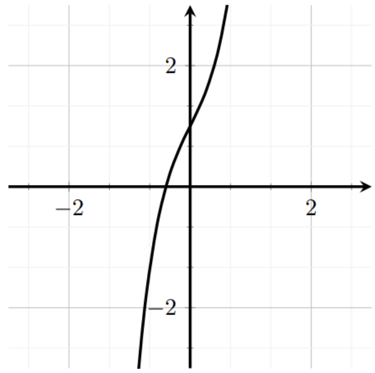
$$f^{-1}(y) = y^5 + 3y^3 + 2y + 1.$$

(a) Compute $f^{-1}(1)$ and $f(1)$.

(b) Compute the value of $x_0$ such that $f(x_0) = 1$.

(c) Compute the value of $y_0$ such that $f^{-1}(y_0) = 1$.

(d) Given the graph of $f^{-1}$ below, graph $f$.




5. Let $f\colon A \to B$ be a function. Suppose $f$ has an inverse. Prove that the inverse is unique *Hint: assume $g, h$ are both inverses of $f$ and show that $g = h$.*

# 3. Limits

1. (SFU) Sketch the graph of an example function that satisfies the following conditions:

$$\lim_{x\to 0} f(x) = 1, \quad \lim_{x\to 3^-} f(x) = -2, \quad \lim_{x\to 3^+} f(x) = 2, \quad f(0) = -1, \quad f(3) = 1.$$

2. (SFU) Is there a number $b$ such that

$$\lim_{x\to 2} \frac{bx^2 - 10x + 10 + b}{x^2 - 2x - 2}$$

exists? If so, find $b$ and evaluate the limit.

3. (SFU) Suppose that

$$x^4 < f(x) < x^2$$

if $|x| < 1$ and

$$x^2 < f(x) < x^4$$

if $|x| > 1$. Do the following limits exist? If so, find their value.

(a) $\lim_{x\to -1} f(x)$.

(b) $\lim_{x\to 0} f(x)$.

(c) $\lim_{x\to 1} f(x)$.

4. (SFU) Evaluate the following limits:

(a) $\lim_{x\to 10} \frac{x^2-100}{x-9}$.

(b) $\lim_{x\to 10} f(x)$, where $f(x) = x^2$ for all $x \neq 10$ and $f(10) = 99$.

(c) $\lim_{x\to 8} \frac{\sqrt[3]{x}-2}{x-8}$.

(d) $\lim_{x\to 8} \frac{(x-8)(x-2)}{|x-8|}$.

5. The rigorous definition of limits is the following. Let $f$ be a function defined in an interval containing a point $x = a$ (but possibly not defined at $x = a$). We say that

$$\lim_{x\to a} f(x) = L$$

if for every number $\epsilon > 0$, there is a number $\delta_\epsilon > 0$ such that

$$|f(x) - L| < \epsilon \quad \text{whenever} \quad 0 < |x - a| < \delta_\epsilon.$$

Essentially, $f$ is arbitrarily close to $L$ whenever $x$ is close enough to $a$.

Use this definition to show that $\lim_{x\to 0} x^2 = 0$.

# 4. Continuity

1. (SFU) Define the function $f$ by

$$f(x) = \begin{cases} 2x+1 & x \le 0 \\ bx^3 - 1 & 0 < x \le 1 \\ x^2 + 2b & x > 1 \end{cases},$$

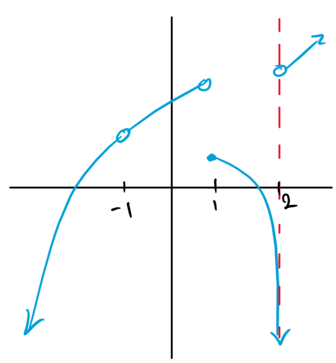
where $b$ is some constant.

(a) Is the function $f(x)$ continuous at $x = 0$? Justify your answer.

(b) Determine the value of $b$ such that $f$ is continuous at $x = 1$.

2. (SFU) Give an example of a function $f(x)$ that is continuous for all values of $x$ except at $x = 3$, where $f$ has a removable discontinuity. Explain how you know $f$ is discontinuous at $x = 3$, and how you know the discontinuity is removable.

3. For the graph below, classify the discontinuities as infinite, jump, or removable.



4. Define a function $f$ by

$$f(x) = \begin{cases} ax+b & x \leq 1 \\ \ln(x) & x > 1 \end{cases}.$$

Find all values of $a, b$ such that $f$ is continuous at $x = 1$.

5. Define a function $f$ by

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}.$$

Show that $f$ is continuous at $x = 0$.

# 5. Intermediate Value Theorem

1. (SFU)

   (a) Show that $2^x = \frac{10}{x}$ has a solution for some $x > 0$.

   (b) Show that $2^x = \frac{10}{x}$ has no solution for $x < 0$.

2. (SFU) Suppose $f$ is continuous on $[1, 5]$ and the only solution of the equation $f(x) = 6$ are $x = 1$ and $x = 4$. If $f(2) = 8$, explain why $f(3) > 6$.

3. Show that the polynomial $P(x) = x^3 - 2x + 1$ has a root, i.e. $P(c) = 0$ for some $c \in \mathbb{R}$.

4. Let $P(x) = a_0 + a_1x + \ldots + a_{n-1}x^{n-1} + x^n$ be a real polynomial ($a_0, \ldots, a_{n-1} \in \mathbb{R}$) of odd degree (i.e. $n \in \mathbb{N}$ is odd). Use the intermediate value theorem to show that $P$ has at least one real root, i.e. $P(c) = 0$ for some $c \in \mathbb{R}$. *Hint: the argument is similar to Problem 5.3 above.*

5. Let $f$ be a continuous function on some interval $[a, b]$. Prove that $f([a, b])$ is also an interval. *Hint: choose $c, d \in f([a, b])$ and write $c = f(x_1)$, $d = f(x_2)$ for some $x_1, x_2 \in [a, b]$. Since $f$ is continuous on $[x_1, x_2]$, IVT can be applied.*

# 6. Limits at Infinity, Indeterminate Forms

1. (SFU) Find an example of a function $g$ such that:

   (a) the line $y = 3$ is a horizontal asymptote of the curve $y = g(x)$, **and**

   (b) the curve $y = g(x)$ intersects the line $y = 3$ at infinitely many points.

2. (SFU) Find the vertical and horizontal asymptotes of the curve

$$y = \frac{1 + x^4}{x^2 - x^4}.$$

3. Fix any positive real number $r > 0$. Show that

$$\lim_{x \to \infty} \frac{\ln(x)}{x^r} = 0.$$

4. Determine whether or not the following are indeterminate forms:

$$1^\infty, \quad \infty^1, \quad \infty^0, \quad 0^\infty.$$

   For the indeterminate forms, give an example of when the limit is 0, 1, and $\infty$.

5. The formal definition of a limit at infinity is the following:

   Given a function $f$, we say that $\lim_{x\to\infty} f(x) = L$ if for every $\epsilon > 0$ there is $N_\epsilon > 0$ such that if $x > N_\epsilon$, then $|f(x) - L| < \epsilon$.

   Use this definition to show that $\lim_{x\to\infty} \frac{1}{x} = 0$.

# 7.